Effects of experimental approach on performance of physical pipe culvert models and implications for use of results at prototype scale

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Sammendrag

Effekter av eksperimentelle fremgangsmåter for fysiske rørkulvertforsøk og betydning for bruk av resultater i fullskala. Rammeverk for hydraulisk dimensjonering av kulverter er i stor grad basert på fysiske modellforsøk. Denne artikkelen tar for seg forskjellige eksperimentelle fremgangsmåter og deres betydning for bruk av resultatene i fullskala. Resultatene viser at tilløpsstrømning, atmosfærisk undertrykk i modellkulverten og effekten av den hydrodynamiske innløpslengden kan påvirke den hydrauliske effektiviteten til fysiske kulvertmodeller i betydelige grad. Sammenligning av eksperimentelle resultater og FHWA-rammeverket viser at dette rammeverket er konservativt både for type 5-strømning og overgang mellom type 5- og 6-strømning. Sammenligning av eksperimentelle resultater og USGS-rammeverket viser at dette rammeverket er konservativt for type 5-strømning men potensielt ikke-konservativt for overgang mellom type 5- og 6-strømning. Forslag til en beste praksis for fysiske kulvertforsøk, inkludert minimumseffektivitet for utløpskontroll, og videre arbeid er beskrevet.

Summary

Hydraulic culvert design frameworks are largely based on the results of physical model experiments. This paper reviews the effects of different experimental approaches on the performance of physical pipe culvert models, and the implications for use of the results at prototype scale. The results show that approach flow conditions, sub-atmospheric air pressure in the culvert model barrel and hydrodynamic entrance length effects can significantly affect culvert model performance. Comparison of experimental results to the Federal Highway Administration (FHWA) framework shows that this framework can be considered conservative for both type 5 flow performance and type 5-to-6 flow transition. Comparison to the U.S. Geological Survey (USGS) framework shows that it can be considered conservative for type 5 flow, but that type 5-to-6 flow transition criteria are simplified or potentially scale dependent. Recommendations for a best practice experimental approach, including minimum outlet control performance, and further work are given.

Notation

A = Cross section area of culvert barrel [m^2]
A_a = Approach cross section area of flow [m^2]
A_b = Blockage cross section area [m^2]
A_f = Cross section area of flow at inlet face [m^2]
A_m = Cross section area of culvert model barrel [m^2]
c = Type 5 flow discharge coefficient [s^2/ft]
D = Culvert rise/diameter [m]
D_m = Model culvert rise/diameter [m]
f = Friction factor [-]
f_{app} = Apparent friction factor [-]
g = Gravitational acceleration (9.81) [m/s^2]
H_w = Headwater elevation [m]
H^* = Dimensionless headwater elevation [-]
k_e = Entrance loss coefficient [-]
k_{eb} = Blockage entrance loss coefficient [-]
K = Type 1 flow discharge coefficient [-]
K_s = Slope correction term [-]
K_u = Unit conversion factor (1.811) [ft$^{0.5}$/s$^{0.5}$]
L = Culvert length [m]
L_e = Hydrodynamic entrance length [m]
L_m = Model culvert length [m]
m = Contraction ratio factor [-]
M = Type 1 flow discharge exponent [-]
n = Manning's roughness coefficient [s/m$^{1/3}$]
P_a = Sub-atmospheric air pressure [N/m^2]
Q = Discharge [m^3/s]
Q_b = Discharge (partial blockage) [m^3/s]
Q^* = Semi-dimensionless discharge [ft$^{0.5}$/s]
r = Inlet edge rounding radius [m]
R = Hydraulic radius [m]
R_a = Approach channel Reynolds number ($4vR/\nu$) [-]
R_D = Culvert barrel Reynold number (vD/ν) [-]
S = Culvert barrel slope [m/m]
S_f = Friction slope [m/m]
u_p = Uncertainty interval of parameter p [varies]
v = Average flow velocity [m/s]
w = Inlet bevel edge width [m]
x = Distance from inlet section [m]
Y = Type 5 flow pressure term [-]
ΔH_e = Entrance head loss [-]
η = Control surface orientation [°]
ν = Kinematic viscosity of water [m^2/s]
ρ = Density of water [kg/m^3]
τ_w = Wall shear stress [N/m^2]

Introduction

Culverts are widely used hydraulic structures, used to safely convey water through embankments or other hindrances. The Federal Highway Administration (FHWA) and the U.S. Geological Survey (USGS) frameworks are commonly used for hydraulic culvert design and are therefore the focus of this study. The FHWA framework is mainly used for determining the minimum culvert size required to safely pass a known or assumed design discharge. This framework conservatively determines the highest headwater elevation based on the design of the culvert, discharge and tailwater conditions (Schall et al. 2012). The USGS framework is mainly used to indirectly measure peak discharge based on the culvert flow conditions. This framework determines the discharge based on the design of the culvert, probable flow type and field measurements of upstream and downstream flow conditions (Bodhaine 1968). Both frameworks use empirical design values taken from physical scale model experiments to account for the hydraulic effects of different culvert designs and flow conditions. A significant number of physical model studies have been conducted over the last hundred years (McEnroe 2007). These studies have used different experimental approaches, and the results show variations in hydraulic performance for similar culvert designs. The experimental results also show variation in performance with factors that are not accounted for in the noted design frameworks. These variations and the different implementations of experimental results in hydraulic design frameworks can lead to differences between estimated and actual hydraulic performance at prototype scale.

The objective of this study has therefore been to review the effects of different experimental approaches for physical pipe culvert model experiments on the empirical design values used in hydraulic culvert design frameworks. The goals of this work have been to (1) inform hydraulic engineers and researchers of potential scale effects associated with the hydraulic design values, and (2) suggest a best practice experimental approach for physical culvert model experiments. The hydraulic scaling of culvert barrel roughness is well described in available literature and was therefore not considered in this study. In addition, this study has considered only circular pipe culverts due to the available data for this culvert type.

Culvert hydraulics

Culvert flow conditions

Culvert flow conditions are commonly classified using the USGS classification system, based on the location of the control section and relative submergence of the inlet and outlet (Bodhaine 1968; Schall et al. 2012) (Fig. 1a-f). In addition to these flow types, experimental studies have identified other flow types such as slug flow with gross intermittent air entrainment, and submerged flow under significant influence of air-carrying vortices (Straub et al. 1953; French 1961) (Fig. 1g-h). Detailed descriptions of the different flow types and corresponding hydraulic performance are given in the referenced publications. More broadly, it is common to distinguish between inlet control (IC) and outlet control (OC) operation.

Hydraulic culvert performance

For IC conditions, hydraulic performance is commonly described in semi-dimensionless form (Schall et al. 2012):

Type 1 flow (form 1): $$\frac{H_w}{D} = K\left(\frac{K_u Q}{AD^{0.5}}\right)^M \quad (1)$$

Type 1 flow (form 2): $$\frac{H_w}{D} = \frac{H_c}{D} + K\left(\frac{K_u Q}{AD^{0.5}}\right)^M + K_s S \quad (2)$$

Type 5 flow: $$\frac{H_w}{D} = c\left(\frac{K_u Q}{AD^{0.5}}\right)^2 + K_s S \quad (3)$$

In this paper, eqs. (1) – (3) are collectively referred to as the "H^*-Q^* relationship" ($H^* = H_w/D$ and $Q^* = K_u Q/AD^{0.5}$). For OC conditions, performance is a function of the head at the control section and the head losses between the control section and the approach zone (Schall et al. 2012):

Type 2, 3, 4, 6 and 7 flow: $$\sum \Delta H = \left[k_e + \frac{19.63\, n^2\, L}{R^{1.33}} + 1\right]\frac{v^2}{2g} \quad (4)$$

The empirical parameters in eqs. (1) – (4) (K, M, c, Y, K_s, k_e, n) are based on the results of physical model experiments and are here collectively referred to as "design values". For further

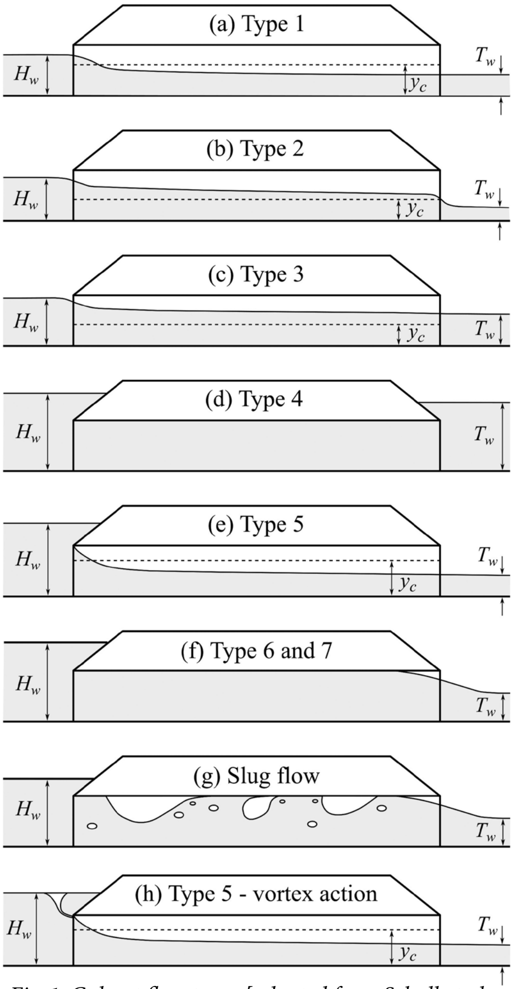

Fig. 1. Culvert flow types [adapted from Schall et al. (2012) and French (1961)].

descriptions of the governing equations and design values used in the USGS and FHWA frameworks, the reader is referred to Bodhaine (1968) and Schall et al. (2012), respectively. For the purpose of this study, efficiency under IC operation was defined by the H^*/Q^* ratio, with a low ratio indicating high efficiency, and OC efficiency was defined by the inlet loss coefficient (k_e), with a low value indicating high efficiency.

Reviewed experimental studies

The reviewed experimental studies and important experimental setup parameters are given in Table 1. The list of studies in Table 1 is not exhaustive but rather chosen to cover a wide range of experimental setups. The reviewed studies show that for physical culvert model experiments, dimensionless or semi-dimensionless Froude scaling is commonly used for IC conditions, and that scaling is not used for OC conditions. In physical hydraulic models, scale effects arise due to differences in the ratios of forces that affect water flow at different scales (Heller 2011). Reduction of these effects to acceptable magnitudes at model scale is therefore necessary for use of the results at prototype scale. As culvert design frameworks are used for culverts of different sizes, a specific scale factor cannot be determined based on the model culvert size (D_m). In the following, the effects of experimental approach on culvert model efficiency are therefore illustrated through comparison of experimental data and design frameworks. For further information about scale effects in hydraulic models and approaches for minimizing them, the reader is referred to Heller (2011).

Effects of experimental setup on model culvert performance

Approach-to-inlet contraction ratio effects

The approach-to-inlet contraction ratio describes the ratio of the inlet cross section area of flow (A_f) to the approach channel cross section area of flow (A_a). Efficiency increases with increasing contraction ratio for both IC and OC conditions, and A_f/A_a varies with H^* and the cross section of the approach channel (French 1955; 1961; Idelchik 1986; Tullis et al. 2008). The USGS design framework accounts for the contraction ratio through the use of an empirical contraction factor ($m = 1 - A_f/A_a$), with mini-

Table 1. Overview of reviewed experimental pipe culvert studies (data from referenced publications).

Study	Flow control	D_m	L_m/D_m	Vented inlets	Blockage	Uncertainty estimates
Shapiro and Smith (1948)	OC	9.5 mm	232	No	No	No
Liskovec (1951)	OC	200 mm	2	No	No	No
Straub et al. (1953)	IC + OC	102 mm	105	No	No	No
French (1956)	OC	140 - 305 mm	95.6	No	No	No
French (1961)	IC + OC	140 - 672 mm	12 - 66	Yes	No	No
Schiller (1956)	IC + OC	127 mm	10.4 - 13.8	No	No	No
Augustine (1988)	OC	15.8 mm	372	No	No	Yes
Smith and Oak (1995)	IC + OC	121 - 201 mm	6.7 – 30.6	No	No	No
Tullis et al. (2008)[a]	IC + OC	230 - 600 mm	16.5 – 26.5	No	Embedment	Yes
Tullis and Anderson (2010)	IC + OC	299 mm	20.4	No	No	Yes
NTNU (2023)	IC	50 mm	10.1	No	No	No
Guerrero (2023)	IC + OC	296 mm	27	Yes	Yes	Yes
Sellevold et al. (2023)	IC + OC	375 mm	21.2	Yes	Yes	Yes

[a] For embedded culverts the vertical rise of the non-embedded part of the inlet was used as *D*.

mum performance for $A_f/A_a \leq 0.20$ (Bodhaine 1968). The FHWA framework does not include contraction ratio effects, but uses a minimum performance approach corresponding to $A_f/A_a \leq 0.20$ (Schall et al. 2012).

Inlet control performance

French (1957;1961) concludes that type 5 flow performance is significantly influenced by approach flow conditions and the air-pressure over the water surface in the inlet. Under these conditions, eq. (3) becomes (French 1961):

$$\frac{H_w}{D_m} = c\left(\frac{K_u Q}{A_m D_m^{0.5}}\right)^2 + Y + K_s S + \frac{P_a}{\rho g D_m} \qquad (5)$$

The sub-atmospheric air pressure (P_a) is measured relative to atmospheric pressure, and a negative value yields increased efficiency. French (1957) found P_a to be a function of air entrainment near the inlet and ventilation through air flow in the barrel and air-carrying vortices over the inlet (Fig. 1h), and that P_a increased with Q^*. P_a can therefore be expected to depend on D_m and L_m, as well as the discharge and approach flow turbulence and flow symmetry (French 1957; 1961). From eq. (5) it can be seen that P_a affects the values of c and Y in eq. (3) and will depend on the specific values of Q^* used in experiments. However, French (1957;1961) found that the use of vented inlets reduced this effect, ensuring largely scale invariant, minimum type 5 performance for models of $D_m \geq 140$ mm.

Fig. 2a shows performance for square edge inlets in headwalls under confirmed type 5 flow operation, showing a span of performance across scales. For the $D_m = 50$ mm model of NTNU (2023), no ventilating vortices were observed, consistent with increased efficiency (French 1957;1961). Fig. 2b shows performance for thin-walled projecting inlets, indicating different flow types for similar culvert designs. The performance of the unvented models in Fig. 2b show a larger span of performance and flow types than those of Fig. 2a, while the vented model of $D_m = 305$ mm closely approximates the minimum performance (adjusted for $P_a/\rho g D_m$), and the FHWA performance. For projecting inlets, performance depends on the inlet wall-thickness, which might explain the difference between the minimum and FHWA performance (French 1961). Not shown in Fig. 2b are the results of Straub et al. (1953) and Smith & Oak (1995), which closely approximate the performance of the FHWA framework for corresponding inlet geometries. These results serve to illustrate the effects of approach flow conditions

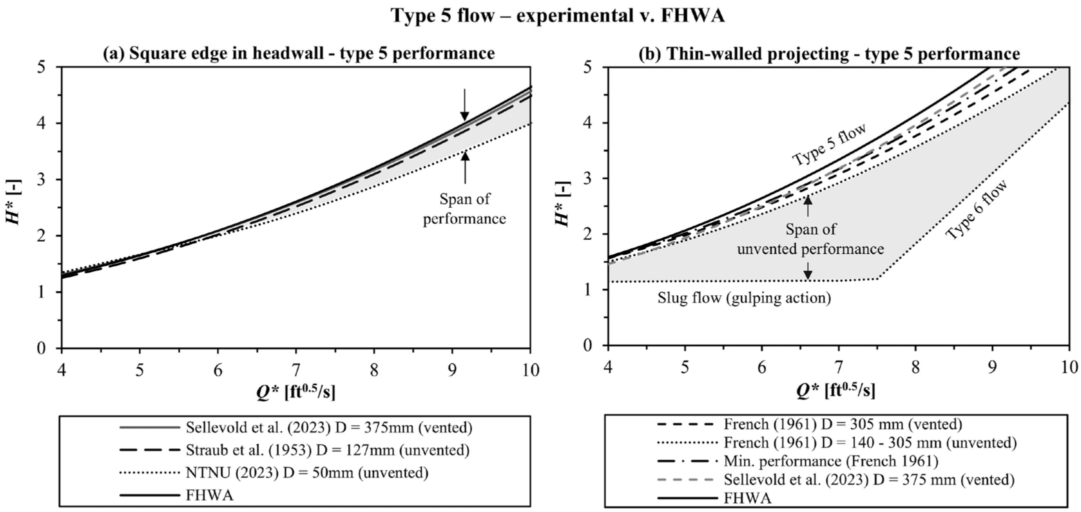

Fig. 2. Comparison of performance (data from referenced publications).

and inlet ventilation, as significant vortex action effectively reduces the magnitude of P_a, similarly to the use of vented inlets. The maximum performance difference relative to that of the FHWA framework is approximately 13% for type 5 flow and 69% for slug and full conduit flow for $Q^* = 7.5$ $ft^{0.5}/s$ (Fig. 2). These results indicate that the use of vented inlets or measurements of P_a for correction of c and Y according to eq. (5) is important for ensuring scale invariant type 5 performance. In relation to the data in Fig. 2, it should also be noted that vented inlets do not fully eliminate the sub-atmospheric pressure ($P_a \neq 0$), as indicated by the difference between the vented and minimum performance in Fig. 2b. The FHWA framework notes the effects of P_a on type 5 performance, but conservatively assumes a minimum performance, consistent with the data of Fig. 2.

Type 5-to-6 flow transition

As indicated by Fig. 2b, both performance and prevailing flow type are associated with scale dependence for unvented culvert models. French (1957) documents different type 5-to-6 flow transitions with initial full conduit flow near either the inlet or outlet, dependent on the approach flow conditions, vortex action, P_a, S, and the model inlet geometry. The USGS framework determines type 5-to-6 flow transition based on L/D, S, n, R, H_w, and the inlet geometry (w/D or r/D) (Bodhaine 1968). Fig. 3 shows the USGS flow type criteria compared to experimental results for type 5 flow. Type 5 and 6 flow are indicated on the left and right sides of each line, respectively. It was found that the results of the unvented models of Schiller (1956) and Straub et al. (1953) were consistent with the determination criteria, but several counterexamples were found in the data of French (1955;1961) and Sellevold et al. (2023) for both vented and unvented models of D_m = 140 – 745 mm (Fig. 3). French (1955;1961) and Sellevold et al. (2023) used smooth walled pipes, associated with delayed onset of type 6 flow, as smaller friction losses reduce the tendency of the water surface to intersect the barrel crown downstream of the inlet (Bodhaine 1968). However, the experimental data in Fig. 3 deviate significantly from the USGS flow type criteria with regards to both inlet geometry and friction effects. It should also be noted that the flow type criteria illustrated in Fig. 3 do not specify the values of H^* or Q^* for which transition occurs, but that both type 5 and 6 flow are valid for approximately $H^* \geq 1.5$ (Bodhaine 1986). The results of French (1961) show examples of performance between type 5 and 6 flow in the range $1.2 \leq H^* \leq 3.6$. These results indicate that type 5-to-6 flow transitions are dependent on a number of factors beyond the USGS flow type criteria. French (1956b) also discusses the interdependency of R_a, H^* and A_f/A_a, and notes a tendency of prevailing type 5 flow at higher values of H^* for lower values of A_f/A_a. To the degree that vented inlets produce scale invariant type 5 flow performance, the results of Fig. 3 indicate that the USGS determination criteria are simplified or potentially scale dependent. For model studies for specific prototype culverts, correct scaling of the approach flow and P_a is therefore important in order to determine the prevailing flow type.

Based on the findings, the dependencies of H^* for type 1 and 5 flow, and type 5-to-6 flow transitions are as follows:

$$\text{Type 1 flow: } H^* = f\,(Q^*, S, A_a/A_f, \text{inlet geometry}) \tag{6}$$

$$\text{Type 5 flow: } H^* = f\,(Q^*, S, P_a, R_a, A_a/A_f, \textit{approach flow symmetry, inlet geometry}) \tag{7}$$

$$\text{Type 5-to-6 flow transition} = f\,(Q^*, H_w, R, n, P_a, R_a, A_a/A_f, \text{approach flow symmetry, inlet geometry}) \tag{8}$$

Outlet control performance

Under OC operation the entrance head loss (ΔH_e) is commonly estimated as the product of the entrance loss coefficient (k_e) and the velocity head (Schall et al. 2012):

$$\Delta H_e = k_e \frac{v^2}{2g} \tag{9}$$

For type 2 and 3 flow, k_e varies with H^* and attains significantly constant values for type 4, 6

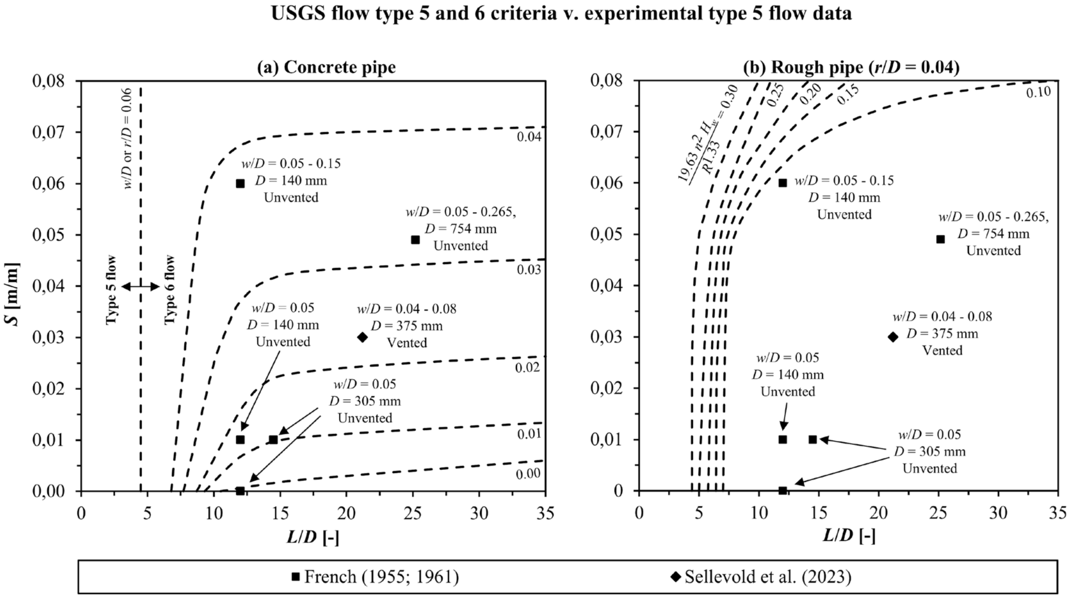

Fig. 3. USGS Flow type criteria v. experimental data of French (1961) and Sellevold et al. (2023) (adapted from Bodhaine 1968 and Chin 2013).

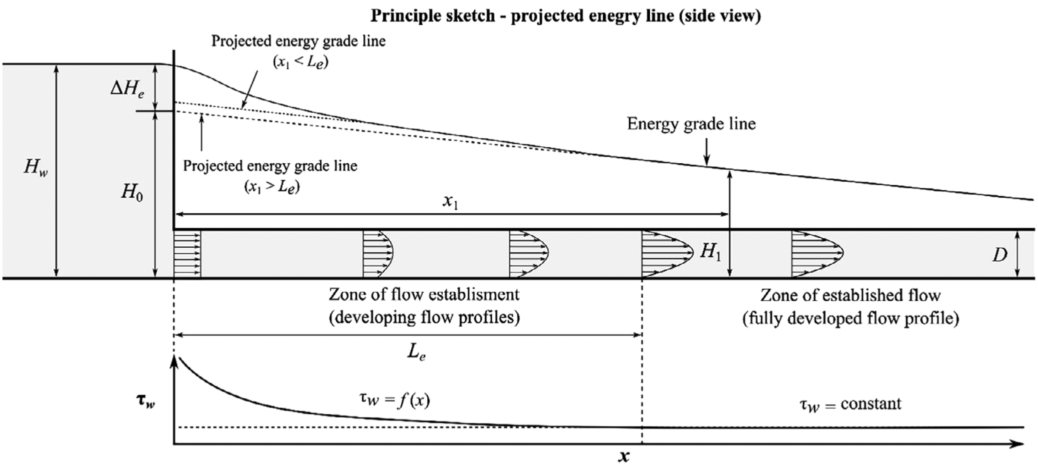

Fig. 4. Principle sketch of projected v. measured energy grade line (adapted from French 1956a and White 2016).

and 7 flow in the range $H^* = 1.2 - 1.5$ (Smith and Oak 1995; Tullis et al. 2008; Tullis and Anderson 2010; Sellevold et al. 2023). In the referenced publications, k_e varies between 0.43 – 0.55 for square edge inlets in headwalls, and 0.80 – 0.98 for thin-walled projecting inlets for type 4, 6 and 7 flow (Liskovec 1951; Straub et al. 1953; Smith and Oak 1995; Tullis et al. 2008; Tullis and Anderson 2010; Sellevold et al. 2023). The FHWA design values for these inlets are $k_e = 0.5$ and 0.9, giving maximum differences of 0.07 and 0.10, respectively.

Experimentally, k_e is commonly determined as the difference between the headwater elevation, and the projected energy line at the inlet section, accounting for friction losses in the culvert barrel (Fig. 4):

$$\Delta H_e = H_w - H_0 = H_w - (H_1 + S_f\, x_1) \tag{10}$$

Under type 4, 6 or 7 flow, the wall shear stress (τ_w) and apparent friction factor (f_{app}) varies with the developing velocity profile over the entrance length (L_e) (Fig. 4). The entrance length (L_e) depends on the turbulence regime, barrel Reynolds number (R_D) and approach flow conditions (Shapiro and Smith 1948; French 1956a; Augustine 1988; White 2016). French (1956a) reports apparent friction factors 15 - 50% higher than those in the zone of established flow (f) for a model of D_m = 305 mm, x_1/D_m = 24 and R_D = 5.0 x 10^5. Shapiro and Smith (1948) reports that f_{app} for a smooth pipe of D_m = 9.5 mm varied between approximately 150% and 50% of f for $x_1/D_m \leq 4$ for 3.8 x $10^4 \leq R_D \leq$ 2.3 x 10^5. The results indicate that f_{app} depends on both R_D and the location of turbulence regime transition relative to the inlet section, giving a large envelope curve for f_{app}/f (Fig. 5). The stated accuracy of the friction factors of Shapiro and Smith (1948) are given as within 0.5 – 1.0% based on the accuracy of the measurements, and the results show differences of approximately 2 – 5% compared to the data of Nikuradse (1932) for fully developed turbulent flow. The same dependencies were found by Augustine (1988), using a model of D_m = 15.8 mm and 5.0 x $10^3 \leq R_D \leq$ 1.5 x 10^4. The small model diameters (D_m = 9.5 – 15.8 mm) indicate that the observed turbulence regime transitions and resulting values of $f_{app}/f < 1.0$ are unrealistic for culverts at prototype scale, as both R_a and R_D increase with scale (French 1956b; Tullis et al. 2008).

The effect of f_{app}/f > 1.0 is an increase in the friction slope (S_f) near the inlet, and corresponding decrease in ΔH_e and k_e with increasing R_D for $x_1 < L_e$ according to eq. (10) (Fig. 4). This trend can be seen in the results of Liskovec (1951), Tullis et al. (2008); Tullis and Anderson (2010) and Sellevold et al. (2023). The results are shown in Fig. 6, along with the difference between maximum and minimum k_e values. Liskovec (1951) used a model of L_m/D_m = 2.0 and determined the discharge coefficient for different efficient inlets, from which k_e was calculated (Chin 2013). Tullis et al. (2008), Tullis and Anderson (2010) and Sellevold et al. (2023) used barrel roughness heights from previous experimental studies to determine f. This approach ignores entrance length effects, but it was found that k_e was significantly constant for type 4 flow, consistent with the FHWA frame-

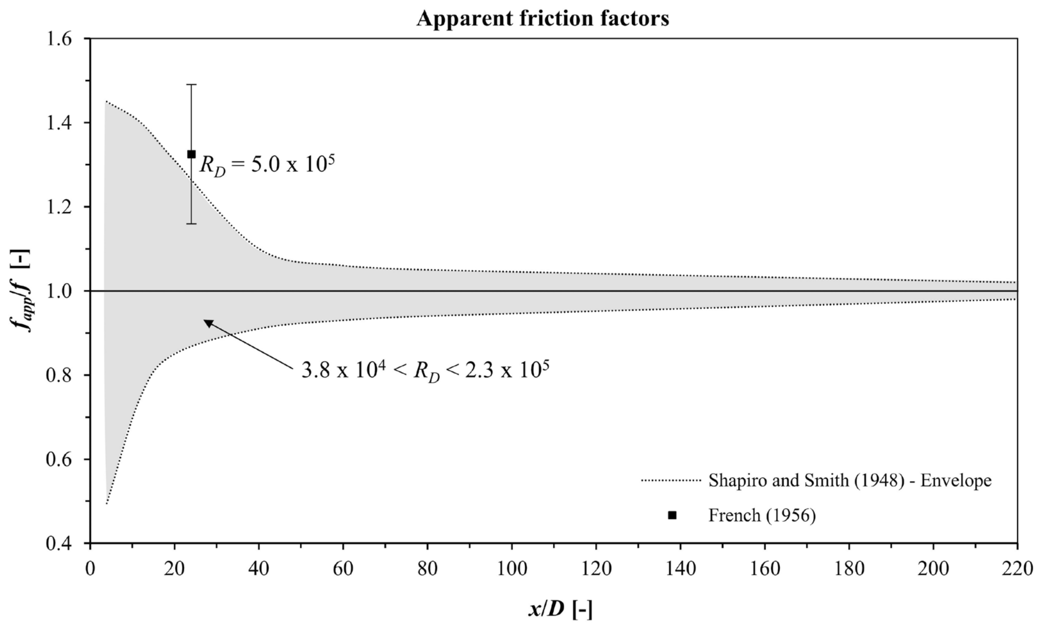

Fig. 5. Span of apparent friction factors (data from referenced publications).

work (Schall et al. 2012). Sellevold et al. (2023) also found that k_e was both highly sensitive to the magnitude of the friction factor and associated with large relative uncertainties for efficient inlets at low values of H^*. Smith and Oak (1995) used model barrels of different lengths to determine the model barrel friction loss and found that the resulting friction slope agreed with friction factors for fully turbulent flow in smooth pipes. However, relatively large variations in k_e were found for $1.2 \leq H^* \leq 1.5$. The results indicate that k_e can be expected to vary with x_1 for $x_1 < L_e$, but that this variation is limited for $R_D \geq 10^4$, $D_m \geq 300$ mm and $x_1/L_e \geq 6$ - 12 (Tullis et al. 2008; Sellevold et al. 2023). A minimum OC performance (analogous to minimum IC performance) can be determined such that k_e accounts for the apparent friction losses in the zone of flow establishment by projecting the energy grade line from a location $x_1 > L_e$ downstream from the inlet (Fig. 4). In planning experiments, L_e can be estimated based on R_D and D_m, or determined experimentally through the use of varying model culvert lengths (L_m).

Based on the findings, the experimental dependencies of k_e are as follows:

$$\text{Type 2 and 3 flow: } k_e = f(H^*, A_a/A_f, \text{inlet geometry}) \quad (11)$$

$$\text{Type 4, 6 and 7 flow: } k_e = f(R_a, R_D, x_1/L_e, A_a/A_f, \text{inlet geometry}) \quad (12)$$

Effects of culvert embedment and inlet blockage

Culverts are prone to blockage of the inlet due to build-up of sediment and floating debris, and culvert embedment is commonly used to ensure favorable flow conditions for aquatic organism passages (Fig. 7). Tullis et al. (2008) reports that inlet embedment has no significant effect on k_e when adjusted for the reduced cross section area of flow, while the H^*-Q^* relationship shows less favorable performance with increasing blockage ratio (A_b/A). Sellevold et al. (2023) reports that inlet blockage causes less favorable performance for both IC and OC conditions with increasing blockage ratio, and that the least efficient performance was generally obtained by mounting a flat plate in front of the face section of the inlet. A comparison of performance under conditions of inlet blockage and culvert embedment is shown in Fig. 8. While based on limited data, the results indicate that a combination of unblocked and partial bottom-up blockage of the inlet using different blockage ratios is sufficient to reasonably estimate the performance of both unblocked, embedded and partially blocked pipe culverts.

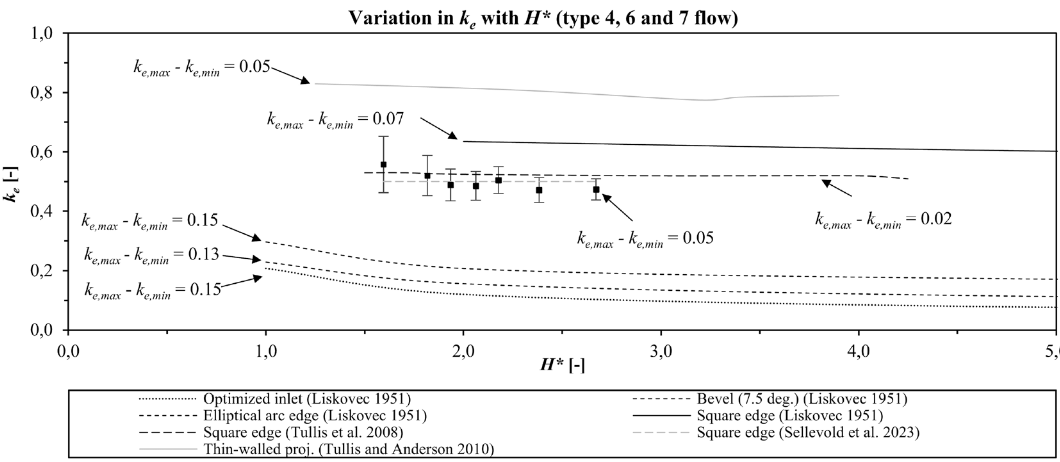

Fig. 6. Variation in ke with H for type 4, 6 and 7 flow (data from referenced publications).*

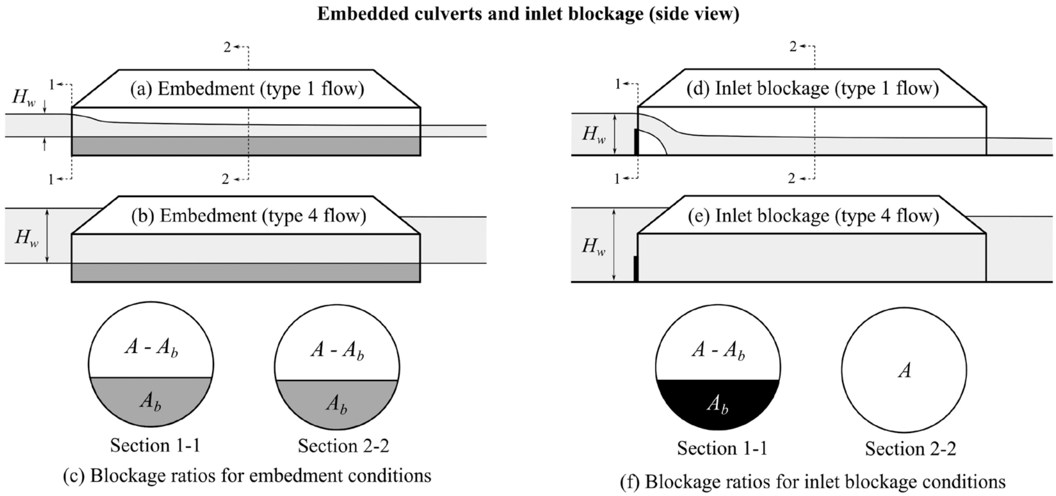

Fig. 7. Embedded culverts and inlet blockage.

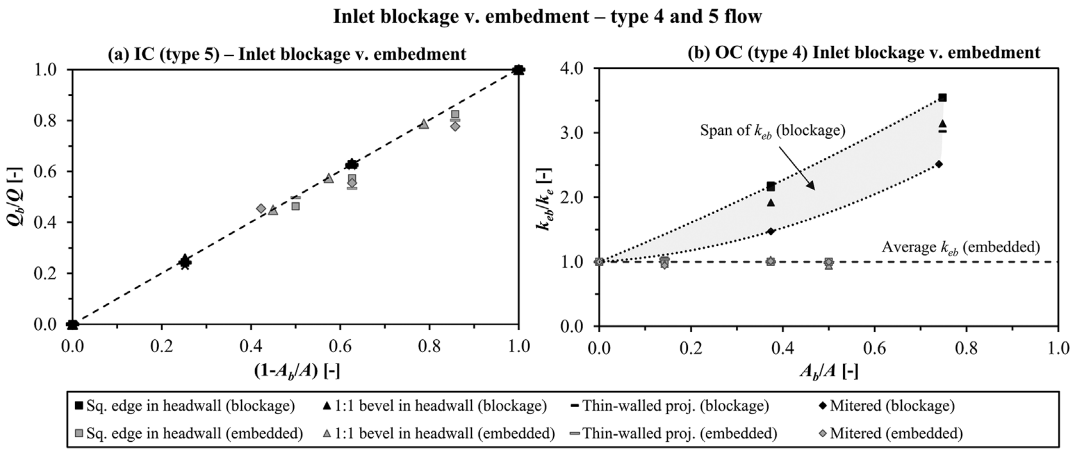

Fig. 8. Hydraulic effects of inlet blockage and embedment (data from Tullis et al. 2008 and Sellevold et al. 2023).

Uncertainty of model culvert performance

The uncertainty of model culvert performance depends on the experimental approach, the accuracy with which model parameters can be determined, and the transient variations of these parameters during the measurements. In the reviewed studies, the method of Kline and McClintock (1953) has been used to determine the uncertainty of k_e, giving the uncertainty of the OC design value directly (Fig. 6), as well as the uncertainty of f_{app} (Augustine 1988; Tullis et al. 2008; Sellevold et al. 2023; Guerrero 2023). From the definition of hydraulic efficiency used in this study, the method can also be extended to the H^*-Q^* relationship as follows: (1) For each measurement, determine the uncertainty of H^* (u_{H^*}) and Q^* (u_{Q^*}). (2) For each measurement, determine the maximum efficiency ($H^* = H^* - u_{H^*}$ and $Q^* = Q^* + u_{Q^*}$) and minimum efficiency ($H^* = H^* + u_{H^*}$ and $Q^* = Q^* - u_{Q^*}$). (3) For the measurement series, determine the design values for the measured, maximum and minimum efficiency through regression. (4) Determine the uncertainty interval of the relevant design values (K, M, c and Y) based on step 3.

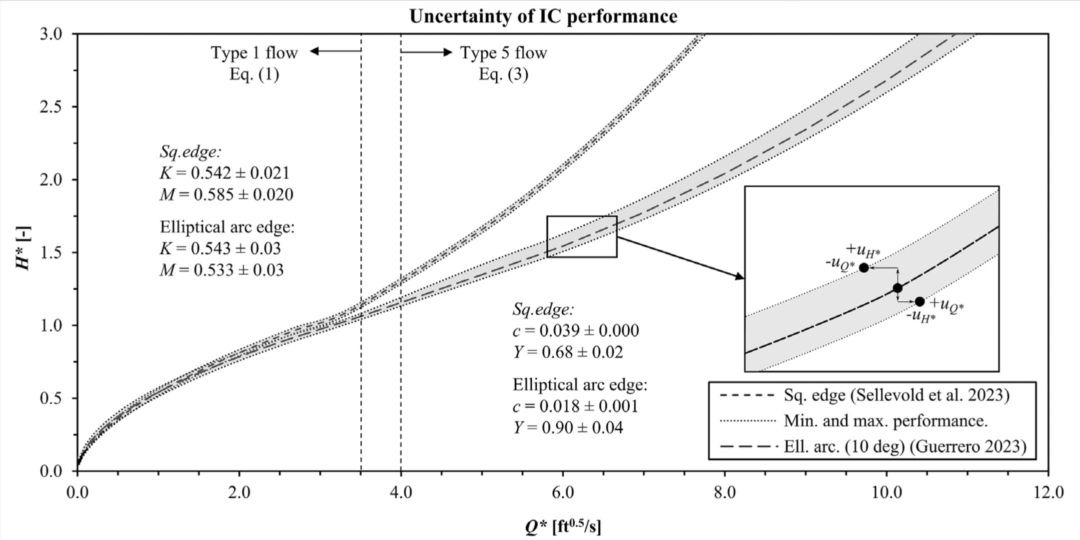

Fig. 9. Uncertainty of K, M (type 1 flow), c and Y (type 5 flow) (data from referenced publications).

The approach is illustrated for the experimental results of Sellevold et al. (2023) and Guerrero (2023) in Fig. 9. The results of the former are for a single experiment, and the results of the latter include maximum variation over three repeated experiments.

The method of Kline and McClintock (1953) does not require repetition of experiments to determine uncertainty but assumes that the uncertainty of each parameter is small compared to the measured value, such that the relationships between uncertainties are approximately linear. The method also only accounts for parameters that are directly involved in the calculation of k_e and the H^*-Q^* relationship. Other factors such as transient variations and dependencies not included in the calculations must be accounted for using other methods (Heller 2011). This is illustrated in Fig. 9, with the results of Guerrero (2023) showing lager uncertainty estimates than those of Sellevold et al. (2023). In addition to direct application to experimental results, the method of Kline and McClintock (1953) can also be used in planning experiments, based on the planned model setup, accuracy of the measurement equipment and hydraulic performance estimated through use of existing design frameworks and/or assumed design values. This approach describes the contribution of the different experimental factors to the uncertainty of the resulting design values, thereby giving a basis for determination of a suitable experimental setup.

Discussion

The results of this study show that both hydraulic performance and flow type transitions depend on a number of hydraulic and pneumatic factors that can be expected to vary between model and prototype scales, as indicated by eqs. (6) – (8) and (11) – (12). The dependencies of unsubmerged and submerged flow are different for both IC and OC conditions. This is reflected in the findings of the reviewed studies, and further work might show further dependencies, e.g. entrance length effects for type 2 and 3 flows. The reviewed studies show experimentally observed flow types that are not included in the FHWA and USGS frameworks. However, the performance of these flow conditions is within the performance of type 5 and 6 flow (Fig. 2b). To the degree that the FHWA and USGS frameworks yield similar performance, they can therefore be considered minimum performance frameworks for type 5 flow. The results also show that type 5-to-6 flow transition is dependent on factors not included in the USGS flow type determination criteria, and comparison to experimental data indicates that the flow type criteria are either significantly simplified or scale dependent. As discussed in Chin (2013), assuming type 5 flow rather than type 6 flow operation is generally conservative for a given value of Q^*. The FHWA framework does not determine the probable flow type but assumes that the flow type resulting in the highest possible headwater elevation will occur (Schall et al. 2012; Chin 2013). This approach therefore excludes the simplifications or potential scale effects related to the USGS flow type transitions and can be considered a minimum performance framework for type 5-to-6 flow transitions.

The FHWA and USGS design frameworks are well established and widely used. In future studies, it is recommended to use an experimental approach that determines the minimum performance for all relevant flow types included in these frameworks, as this allows for direct implementation. Based on available data, nominal symmetrical approach flow and vented inlets of $D_m \geq 140$ mm is sufficient for determination of minimum IC performance, and unvented inlets of $D_m \geq 300$ mm and determination of the energy grade line downstream of the entrance length is sufficient for determination of minimum OC performance. Further work is necessary to determine a lower limit of D_m for both flow control types and flow type transitions. The culvert model size also influences the uncertainty of the results, and this factor should be considered along with the effects of the model setup on hydraulic performance.

For a wider application of the results to partially blocked or embedded culverts, it is also recommended to include blockage effects in future studies. Blockage ratios $A_b/A \approx 0.25$, 0.50 and 0.75 are recommended to determine the relationship between design values and the blockage ratio for the span $0.0 < A_b/A < 1.0$ (Fig. 8). This approach does not require significant modifications to the model setup and is practical in terms of time costs. However, further comparison of embedded culverts and partly blocked inlets is warranted, as the similarity of these conditions is indicated by limited data for circular pipe culverts.

The lack of quantified design value uncertainty is a source of uncertainty in the present study as the significance of the differences in performance between different studies cannot be readily established. However, the FHWA framework gives k_e with one significant digit (i.e. a maximum rounding difference of ± 0.05) and distinguishes between inlets associated with differences in the H^*/Q^* ratio of 2.3% for type 5 flow. Based on this, the noted differences in both OC and IC performance between the reviewed studies can be considered significant for the purpose of practical application at prototype scale. It is therefore recommended to include estimates of the design value uncertainty in future studies for use at prototype scale. Another source of uncertainty in this study is undocumented model parameters in the referenced studies, such as pipe thickness and model culvert length. It is therefore recommended to document all hydraulic and model geometry parameters in future studies. It is also recommended to include culverts with a square edge inlet in a headwall, as this inlet has a simple geometry and only one applicable control surface orientation ($\eta = 90^\circ$) which makes it well suited for benchmark comparison (French 1961).

Conclusions and further work

In this study, different experimental approaches for physical culvert model experiments have been reviewed, and the resulting hydraulic performances have been compared and analyzed in the context of application at prototype scale. Based on the results, the following conclusions are made:

- All USGS culvert flow types are dependent on hydraulic or pneumatic effects not included in the present FHWA and USGS design frameworks, but scale effects can be minimized using experimental minimum performance approaches.

- The FHWA framework can be considered a minimum performance framework for both type 5 performance and type 5-to-6 flow type transition. The USGS framework can be considered a minimum performance framework for type 5 performance, but type 5-to-6 flow transition criteria are not consistent with experimental results in all cases, indicating potential non-conservative simplification or scale dependence.

- For type 5 flow, both performance and transition to type 6 flow depend on sub-atmospheric air pressure in the culvert inlet and approach flow conditions. A significantly scale invariant type 5 flow performance can be determined through the use of vented inlets and nominally symmetrical approach flow for $D_m \geq 140$ mm and $L_m/D_m \leq 66$.

- For type 4, 6 and 7 flow, k_e is dependent on approach flow conditions and apparent friction losses in the zone of flow establishment. Significant scale invariance of k_e has been found for $D_m \geq 300$ mm, $R_D \geq 10^4 - 10^5$, and measurements of the energy grade line more than 6 - 12 D_m downstream from the inlet. Minimum OC performance is obtained through measurement of the energy grade line in the zone of established flow.

- The use of thin-walled blockage plates of varying blockage ratios mounted in front of the inlet can extend experimental results to cases of inlet blockage and culvert barrel embedment under IC and OC conditions.

- The method of Kline and McClintock (1953) or similar methods can be used to determine the uncertainty of experimentally determined design parameters for both IC and OC conditions. Uncertainty estimates and full descriptions of all relevant factors that influence culvert performance are not included in all reviewed studies, and this is a source of uncertainty in the present study.

This study has illustrated a number of hydraulic and pneumatic effects that can cause significant scale effects in physical culvert experiments. The results can be of use to hydraulic researchers and engineers who use the FHWA and USGS design frameworks. The list of reviewed experimental studies included in this paper is not exhaustive, and further analysis could show further simplifications or scale dependencies. For implementation of experimental results at prototype scale, it is recommended that future studies account for the dependencies noted in this paper and use experimental minimum performance approaches consistent with existing design frameworks or specific prototype conditions.

The findings and conclusions of this study can be considered a suggestion for a best practice for physical culvert model experiments. Further experimental work focusing on the scaling of type 5 flow performance, type 5-to-6 flow transitions, determination of scale invariant minimum model culvert diameters and similarities between inlet blockage and embedment is warranted.

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